Supplemental Material: Doping dependence of the magnetic excitations in $La_{2-x}Sr_xCuO_4$

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ALTERNATIVE FITTING ROUTINE

Lamsal *et al* recently argued that the fitting routine used to model the paramagnons in several publications to this point [1–4] does not properly account for the case of critical damping, where the damping factor is greater then double the undamped frequency [5]. In this case, the assignment of the magnon oscillation frequency to the peak center is incorrect as there is not an entire oscillation present. They propose a general equation that is applicable for all cases

$$S(\boldsymbol{Q},\omega) = \frac{\omega\chi_Q}{1-e^{-\beta\hbar\omega}}\frac{2z_Q f_Q}{(\omega^2 - f_Q^2)^2 + (\omega z_Q)^2}, \tag{1}$$

where $S(\boldsymbol{Q},\omega)$ is the scattering intensity, χ_Q represents the strength of the resonance, z_Q is the damping factor, and f_Q is the undamped frequency. From this form, for each $\boldsymbol{Q}$-point, χ_Q modulates the intensity, while z_Q and f_Q effect the position, lineshape, and, to a lesser extent for $f_{\vec{q}}$, the intensity. Applying this fitting routine to our data gave the results displayed in Fig. S1.

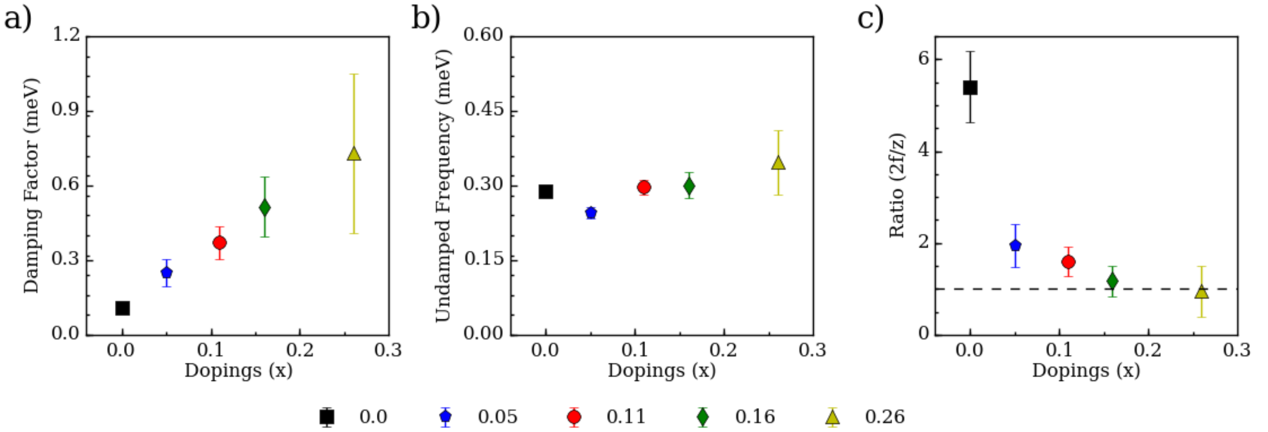

FIG. S1. $\boldsymbol{Q}$-averaged a) z_Q and b) f_Q for each doping used in the manuscript. c) The ratio of twice the undamped frequency to the damping factor for each doping. The condition for critical damping is shown by a dashed line.

Inspection of these results clearly shows the damping factor increases significantly with doping, as was concluded based upon our fitting routine in the main text. The undamped frequency is nearly constant, reflecting the expected magnon dispersion in undoped LSCO. Most importantly, Fig. S1(c) shows that only the 0.26 doping case lies within the critically damped region and is still quite close to 1. Thus, for all other dopings the fitting routine used is perfectly valid, though as Lamsal *et al* argued, for the cases near this line, 0.16 doping, the error from our fitting routine may not be entirely accurate as the peak center may not exactly coincide with the magnon oscillation frequency. In the case of 0.26, we find this fitting routine gives quite large error, as evidenced by the error bar in Fig. S1(a), likely due to a combination of the resolution (~ 120 meV) and the previously mentioned entanglement of the fitting parameters in this method. Thus, we conclude that the successful application of this method to find more accurate values for ω_Q will require much higher resolution and will likely still be difficult in the highly doped samples where the intrinsic broadening of the magnon feature will continue to smear out the features. In this case, both fitting routines are consistent with the same conclusions as described in the main text, i.e. the paramagnons become increasingly damped with doping along the nodal direction. We consequently re-use the previous form in order to better facilitate comparison with previous studies.

It is important to note, the fitting procedure was modified for the doped samples by removing the phonon and multi-magnon peaks, as the broadening of the paramagnon makes them indistinguishable from the paramagnon feature. Thus, the absolute values of the spectral weight and, to a lesser extent, the peak positions are likely slightly offset. However, the offsets between samples should be similar and, as our main concern is the evolution of these features with doping, should not have a significant impact upon our conclusions.

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