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# Note on the definition of fluence in optical experiments

A Gaussian beam has an intensity profile

$$I(r) = I_0 \exp\left(-2\frac{r^2}{w^2}\right) \tag{1}$$

where $r$ is the radial coordinate and $w$ is the $1/e^2$ radius sometime called the waist of the beam. The total energy in such a pulse within radius $r$ can be computed via an integral

$$E(r) = I_0 \int_0^r \exp\left(-2\frac{r'^2}{w^2}\right) \mathrm{d}r' = I_0 \pi \frac{w^2}{2}\left\{1 - \exp\left(-2\frac{r^2}{w^2}\right)\right\}. \tag{2}$$

The large $r$ limit shows that the total energy is

$$E_0 = I_0 \pi \frac{w^2}{2}. \tag{3}$$

The average fluence with radius $r$ is then

$$F(r) = \frac{E(r)}{\pi r^2} = \frac{E_0}{\pi}\frac{1}{r^2}\left\{1 - \exp\left(-2\frac{r^2}{w^2}\right)\right\}. \tag{4}$$

We want to work in terms of the more common peak size metric the full-width at half-maximum (FWHM), which is $X = \sqrt{2\ln(2)}w$ giving

$$F(r) = \frac{E_0}{\pi}\frac{1}{r^2}\left\{1 - \exp\left(-4\ln(2)\frac{r^2}{X^2}\right)\right\}, \tag{5}$$

as shown in Figure 1.

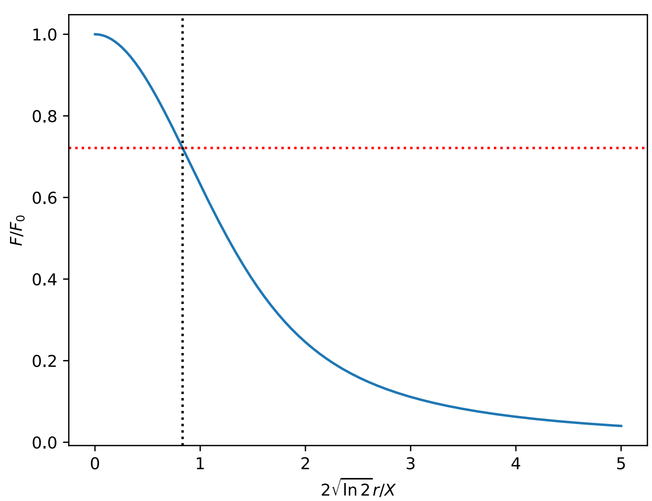


Figure 1: Average fluence, $F$ of Gaussian pulse within radius $r$ in units of the FWHM. The vertical black dotted line corresponds to $r = \frac{X}{2}$ and the horizontal red line is $F = \frac{F_0}{2\ln(2)}$.

At the center of the beam, the peak fluence can be derived by expanding the exponential in a Taylor series

$$F_0 = \frac{4\ln(2)E_0}{\pi X^2}. \tag{6}$$

The average fluence within the FWHM is

$$F(r = X/2) = \frac{2E_0}{\pi X^2} \tag{7}$$

which is the standard form for the fluence equation and which corresponds to $\frac{1}{2\ln(2)} \approx 72\%$ of the peak fluence.